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A new method to estimate wave height of specified return period*

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Abstract In this paper, we propose a new method to estimate the wave height of a specific return period based on the Hurst rule and a self-affine fractal formula. A detailed description of our proposed model is presented in this paper. We use the proposed model to analyze wave height data recorded along the coast of Chaolian Island from 1963 to 1989. The results show that the performance of our proposed model in estimating design wave heights is superior to traditional models.

Keyword: Hurst rule; self-affinity; fractal formula; wave height of specific return period

1 INTRODUCTION

The selection of appropriate design parameters based on marine conditions is of paramount importance in the fields of offshore engineering, coastal engineering, and coastal disaster prevention, in order to estimate the return periods of waves and successfully issue early warnings for storms (Huseby et al., 2013; Rajabalinejad and Demirbilek, 2013). In the last few decades, the annual extreme value (AEV) method has been widely used to estimate the wave height of a specified return period in marine engineering, hydraulic engineering, and the construction of coastal nuclear power plants (Muir and El-Shaarawi, 1986). Domestic hydraulic researchers prefer the Gumbel, Weibull, and Pearson type III distributions because of their successful applications to engineering. However, these have an obvious disadvantage in that they are all based on transcendental estimation. That is, while estimating using these distributions, a prescribed statistical distribution curve is assumed at the outset, and curve fitting for the in-situ annual extreme wave heights is adopted. The curve is then determined and extended in order to estimate the design wave height of a specified return period (Wang et al., 2010a, b, 2011, 2012, 2013). Although this sort of method can pass hypothesis tests, none of these distributions hold in all situations for estimating marine engineering design parameters because of varying geographical and meteorological conditions.

Ocean waves are complicated natural phenomena. The primary technique initially used to study the statistics of ocean waves was the random wave theory proposed by Longuet-Higgins. Following subsequent development, the classical theories of ocean wave height statistics were established. Mei et al. (1995) analyzed ocean wave height series and found that the independence assumption regarding waves was not satisfied. Nevertheless, they found that wave height series are in a long-term correlation and statistically resemble self-affine fractals. Therefore, they

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developed the Cauchy statistical model and the fractal statistical model. However, the self-affinity and scale-invariance of complex ocean waves can only be statistically analyzed. Hence, in this study, we will discuss the fractal properties of ocean waves through a statistical analysis.

The method proposed in this paper is different from traditional methods, which assumes that wave heights satisfy a specific theoretical distribution in advance. However, the method proposed here estimates the wave height of a specified return period by using a self-affine fractal formula based on the Hurst rule from the point of view of fractal theory. After detailing this theory and describing the method of using the formula, we test our method using in-situ data recorded along the coast of Chaolian Island from 1963 to 1989 and compare it with traditional methods. The results show that the design wave height obtained using our method is more accurate than that obtained using traditional methods when the return period is large. Considering the geographical conditions of the island and the effects of typhoon during summer, the wave heights of specific return periods obtained using extreme values during summer deviates from those obtained using annual extreme values as maximum as 1.7%. This shows that our proposed method is more advantageous than traditional methods.

2 THEORETICAL BASIS

2.1 Hurst rule

In 1951, British hydrologist Hurst (1951) found that most natural processes follow the rule of a "random walk with deviation" after he studied a series of in-situ data. He later proposed an empirical formula called the Hurst rule, which can be described as follows. Considering a time series $\{x(t)\}$, t=1, 2, ···, there is an empirical relation for any arbitrary positive integer τ such that

$$R/S \sim \tau^{\mathrm{H}}, \tag{1}$$

where H is a constant, called the Hurst index, in the range [0,1], R and S are the extreme value and mean square deviation, respectively. They are defined as

$$R(\tau)=\max D(t,\tau)-\min D(t,\tau),\quad 1\le t\le\tau, \tag{2}$$

$$S(\tau)=\left\{\frac{1}{\tau}\sum_{t=1}^{\tau}\left\{x(\mathrm{t})-\overline{x(\tau)}\right\}^{2}\right\}^{\frac{1}{2}}. \tag{3}$$

In the above, $D(t, \tau)$ is the accumulated deviation expressed as follows:

$$D(t,\tau)=\sum_{\mu=1}^{t}\left\{x(\mu)-\overline{x(\tau)}\right\},\quad 1\le t\le\tau, \tag{4}$$

where μ is the time parameter. In Eqs.3 and 4 above, $\overline{x(\tau)}$ is the mean value series

$$\overline{x(\tau)}=\frac{1}{\tau}\sum_{t=1}^{\tau}x(t),\quad \tau=1,2,\cdots \tag{5}$$

It is understood from Eqs.2 and 3 that the statistical parameter R/S is dimensionless and empirically obtained. Therefore, the Hurst rule expressed by Eq.1 correlates the results obtained using different time scales τ, which makes it possible to study statistics on a large timescale based on small timescale statistics.

The Hurst index has been widely used to measure the correlation among data series and analyze their tendency. If H=1/2, it is a random walk analysis; If 1/2<H<1 or 0<H<1/2, the tendency of the data series is enhanced or weakened, and it is a random walk with deviation analysis. The Hurst rule (Zhao et al., 2001; Huang, 2005; Ran et al., 2009; Zhang, 2010) has been extensively applied to study the correlation and tendency of time series in hydrology, geology, stocks, and finance. It is also referred to as rescaled range analysis, or R/S analysis.

2.2 Self-affine fractal formula

Brownian motion is a commonly used statistical model for studying random processes. Fractal Brownian motion was first suggested by Mandelbrot in 1968 (Mandelbrot and Van Ness, 1968). The difference between the two models is that the former is independent of increment whereas the latter is dependent on increment. It is easy to obtain the correlation function between past and future increments for fractal Brownian motion (Chu, 2004), which is expressed as

$$c(t)=2^{2\mathrm{H}-1}-1, \tag{6}$$

where H is a constant in the range [0, 1].

It is easy to gather from Eq.6 that if H=1/2, $c(t)$=0: the past and future increments are unrelated, and the motion in question is Brownian with independent increments. If H≠1/2, $c(t)$≠0: in this case, the motion at hand is fractal Brownian. When 1/2<H<1, $c(t)$>0 or 0<H<1/2, $c(t)$<0, and the future and past increments are positively or negatively correlated, respectively. Using these observations, Mandelbrot (Mandelbrot and Van Ness, 1968) proved that fractal Brownian

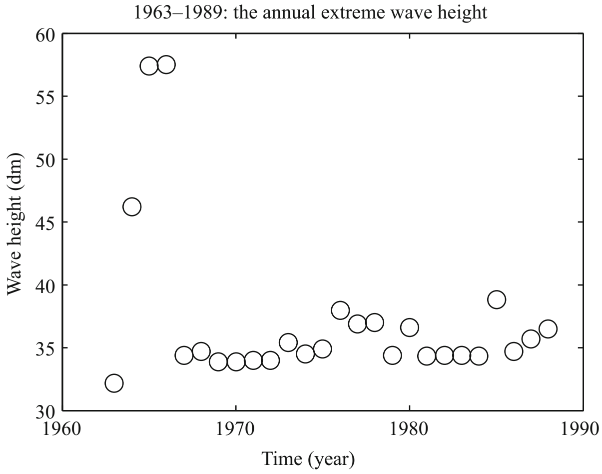

Fig.1 Annual extreme wave height scatter diagram

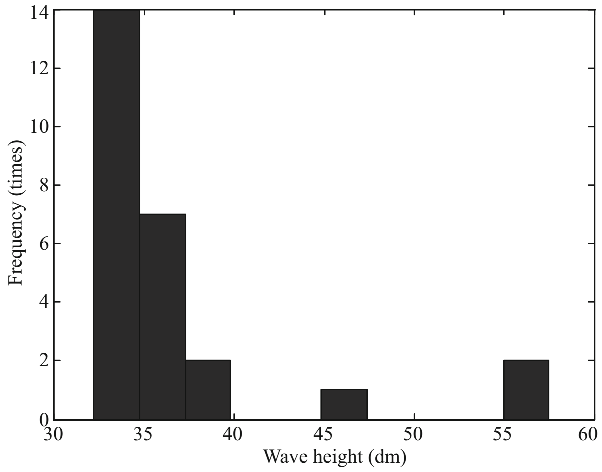

Fig.2 Empirical distribution histogram

motion satisfies the Hurst rule.

In fractal sciences, self-affine statistical fractal (Li and Wang, 1993) requires that

$$f(br)=b^{H}f(r), \tag{7}$$

where $f(r)$ represents the characteristics of the subject.

Fractal Brownian motion satisfies the requirements of a self-affine statistical fractal (Mandelbrot and Van Ness, 1968), which claims that a random function $B_H(t)$ of time in the range [0, L] has the following properties:

$E(B_H(t))=0$,

$\mathrm{Var}(B_H(t))\sim L^{2H}$,

$D(B_H(t))\sim L^{H}$,

where H is a constant in the range [0, 1].

If the mean square deviation $D(B_H(t))$ is selected to describe the statistical characteristics of fractal Brownian motion, then

$$f(bt)=D(B_H(t))\sim b^{H}f(t), \tag{8}$$

This is equivalent to Eq.7. Therefore, fractal Brownian motion satisfies the requirements of a self-affine statistical fractal. Furthermore,

$$P(r)\sim r^{-D}, \tag{9}$$

where $P(r)$ is a characteristic quantity and r is the increment of the variable.

As $P(r)$ is the probability that $x\geq r$ and $P(r)\leq 1$, the exponent in the above equation is negative.

In practice, $P(r)$ can be replaced by the occurrence probability of the incident. If $N(\geq r)$ represents the number of incidents or sets no less than r, where N is the total number of incidents or sets, $N(\geq r)/N$ for accumulating rate and describing the statistical properties of wave height time series, then

$$\frac{N(\geq r)}{N}=\frac{\mathrm{C}}{r^{D}}. \tag{10}$$

In Eq.10, C is a constant and D is the number of dimensions of the fractal set. As a consequence, it becomes relatively easy to estimate the wave height of a specified return period.

As discussed above, we chose fractal Brownian motion as the model to estimate annual extreme waves, which is consistent with the Hurst rule and satisfies the fractal Eq.9. Together, these form the theoretical basis for our new method.

3 PROCEDURES FOR ESTIMATING WAVE HEIGHTS OF SPECIFIED RETURN PERIODS

3.1 *R*/*S* analysis of annual extreme wave height

We analyzed the in-situ wave height data collected from the coastal Nuclear Plant from 1963 to 1989 (the data for 1976 is missing). The measured mean extreme wave height was 3.727 m, and the scatter diagram is shown in Fig.1, where the year 1963 is considered to be the starting point. It shows that the distribution of the annual extreme wave height has sharp peaks and a long tail, which differs from a Gaussian distribution.

Figures 2 and 3 qualitatively indicate that the wave height time series is a random process with tendency. Furthermore, in order to study the fractal of the wave height time series, we analyzed the data by using the rescaled range (R/S) method. We first split the data for 26 years from 1963 to 1989 into the following equally spaced subsections: extreme values every six months, for each year, and for every two years. The wave height time series was then defined as a random process or a process with tendency by examining the

Hurst index (see Table 1). The Hurst index is calculated using the least square method, which takes the Eq.1 logarithmic.

From the data, we can conclude that:

(1) Figure 4 shows that the fitting results of all three groups of data were satisfactory and R^2 was always greater than 0.95, which indicates that the wave height time series satisfies Eq.1: the Hurst rule. Thus, *R*/*S* analysis is applicable.

(2) When the timescale is altered, we can see that 1/2<H<1 and $c(t)>0$ in all three cases (Table 1). According to Eq.6, the wave height time series are long-range correlated.

Following this, we only chose annual extreme values from 1963 to 1989 for *R*/*S* analysis. If 1963 is taken as the starting point of the time series $x(t)$, the value of $R(\tau)$, $S(\tau)$, and $R(\tau)/S(\tau)$ can be derived, as listed in Table 2.

The explicit form of the expression for *R*/*S* and the value of H can be calculated using the method of least squares, i.e., H=0.803 3, $c(t)$=0.522 7, and R^2=0.982 4.

As the Hurst index is 0.803 3 and $c(t)$=0.522 7>0, and given that the closer the Hurst index is to 1, the stronger the long-range correlation, this implies that

Table 1 Data for rescaled range (*R*/*S*) method analysis

	Hurst Index	$c(t)$	R^2
Every 6 months	0.836 9	0.595 2	0.974 5
Every year	0.803 3	0.522 7	0.982 4
Every 2 years	0.828 2	0.576 1	0.976 3

Table 2 Annual extreme values from 1963 to 1989 for *R*/*S* analysis

τ	Year	$R(\tau)$	$S(\tau)$	$R(\tau)/S(\tau)$
1	1963			
2	1964	7.000 0	0.700 0	10.000 0
3	1965	13.066 7	10.309 0	1.267 5
4	1966	18.250 0	10.381 1	1.758 0
5	1967	24.480 0	10.827 7	2.260 9
6	1968	29.900 0	10.678 0	2.800 1
7	1969	34.114 3	10.467 6	3.259 0
8	1970	37.275 0	10.180 6	3.661 4
9	1971	39.700 0	9.866 9	4.023 6
10	1972	41.640 0	9.559 5	4.355 9
11	1973	42.845 5	9.202 7	4.655 8
12	1974	44.075 0	8.915 2	4.943 8
13	1975	45.023 1	8.635 1	5.214 0
14	1977	45.171 4	8.322 9	5.427 4
15	1978	45.520 0	8.052 5	5.653 0
16	1979	45.806 3	7.805 5	5.868 5
17	1980	46.517 6	7.631 6	6.095 4
18	1981	46.783 3	7.425 6	6.300 3
19	1982	47.384 2	7.277 3	6.511 2
20	1983	47.910 0	7.134 1	6.715 6
21	1984	48.385 7	6.998 2	6.914 0
22	1985	48.831 8	6.871 1	7.106 8
23	1986	48.652 2	6.726 0	7.233 5
24	1987	49.000 0	6.607 8	7.415 5
25	1988	49.200 0	6.482 5	7.589 7
26	1989	49.292 3	6.358 5	7.767 9

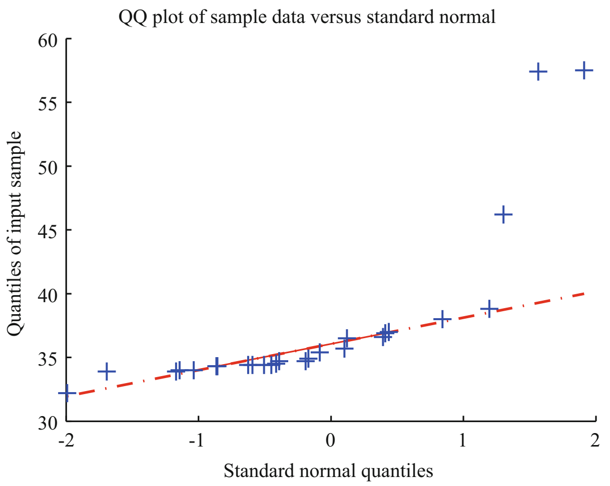

Fig.3 QQ plot of standard normal distribution

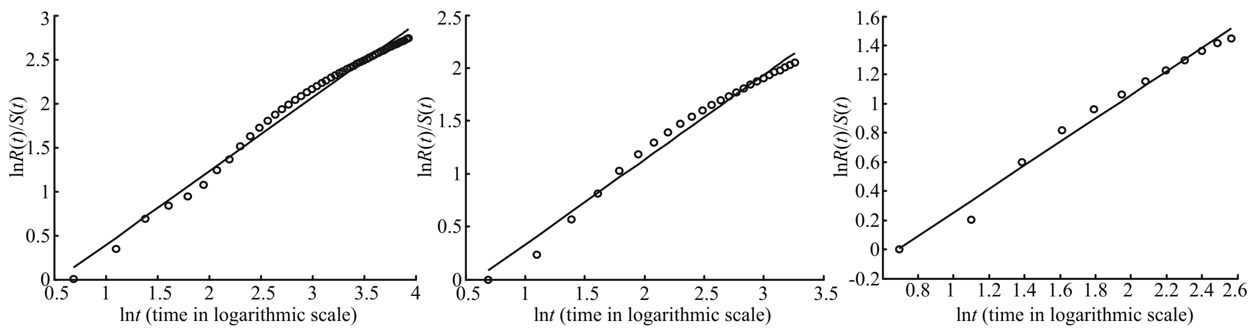

Fig.4 *R*/*S* analysis diagram

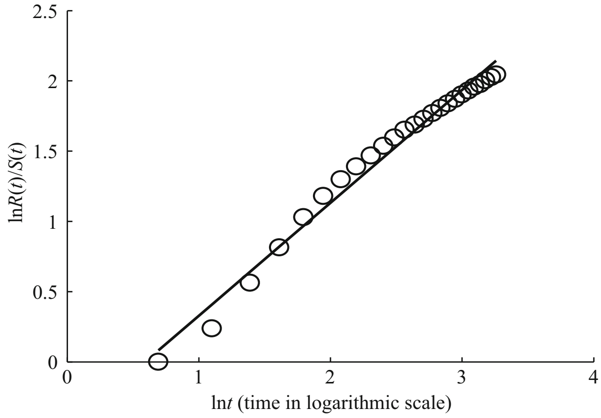

Fig.5 *R*/*S* analysis diagram

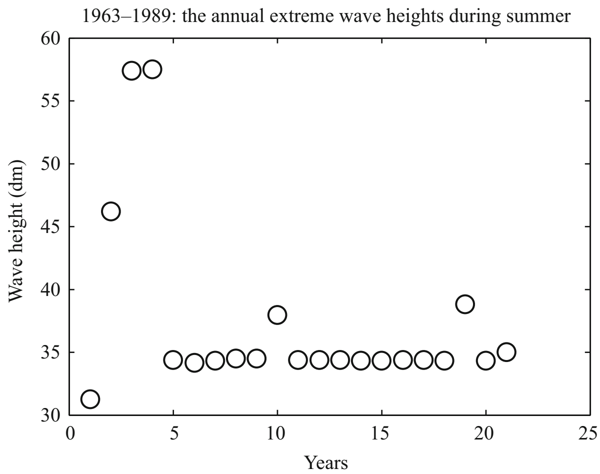

Fig.6 Annual extreme wave height during summer scatter diagram

the wave heights along the coast of Chaolian Island have long-term memory. Furthermore, from an average point of view, as fractal Brownian motion is persistent, the increasing tendency of the height of waves in Chaolian Island in the past corresponds to a similar increasing tendency in the future, and vice versa. Moreover, this implies that the wave height data along the coast of Chaolian Island is non-random. Both R^2=0.982 4 and F=128.31 imply that R/S analysis can satisfactorily fit annual extreme values. The extreme values from 1963–1989 in logarithmic coordinates are shown in Fig.5.

We then analyzed extreme values in July and August in the summer and compared these to the annual extreme values. The scattering diagram is shown in Fig.6.

The explicit form of the expression for R/S, as well as the value of H, can be calculated using least squares method, i.e., H=0.842 9, $c(t)$=0.608 9, and R^2=0.986 8. The Hurst index was 0.842 9 and $c(t)$=0.608 9>0, and as the closer the Hurst index is to 1, the stronger the long-range correlation, we concluded that extreme

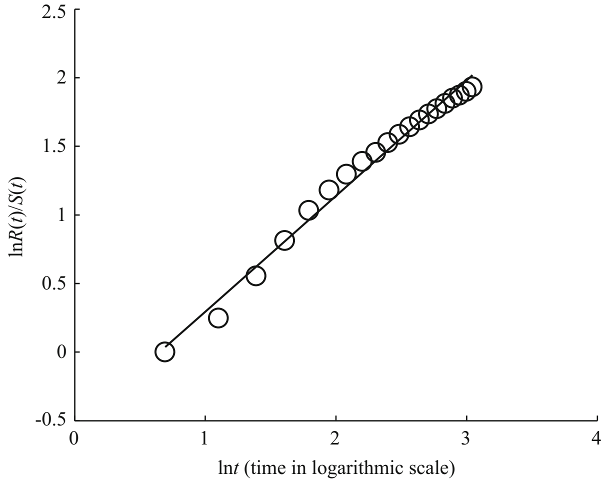

Fig.7 *R*/*S* analysis diagram

Table 3 Parameters of the new model

Data	Parameters	
	C	*D*
Extreme values for 26 years	0.622 1	1.196 7
Extreme values for 13 years	0.499 8	1.064 4

Table 4 Parameters of the new model

Data	Parameters	
	C	*D*
Extreme values in July and August for 26 years	0.577 3	1.154 8
Extreme values in July and August for 13 years	0.506 4	1.072 3

values during summer have long-term memory. Furthermore, from an average point of view, as fractal Brownian motion is persistent, an increasing tendency of the extreme wave heights in past summers corresponds to a similar increasing tendency in the future, and vice vasa. Moreover, this implies that wave height data along the coast of Chaolian Island is non-random in the summer. The correlation coefficient R^2=0.986 8 indicates that R/S analysis can satisfactorily fit extreme values in the summer.

3.2 Comparisons between the fractal model and traditional models

We first split the data into groups as follows:

(1) The extreme values for the 26 years from 1963 to 1989 and those for the first 13 years were selected.

(2) The extreme values in July and August for the 26 years from 1963 to 1989 and those for the first 13 years were chosen.

The parameters (Jiang et al., 2004) derived using the data in groups (1) and (2) are presented in Tables 3 and 4, respectively.

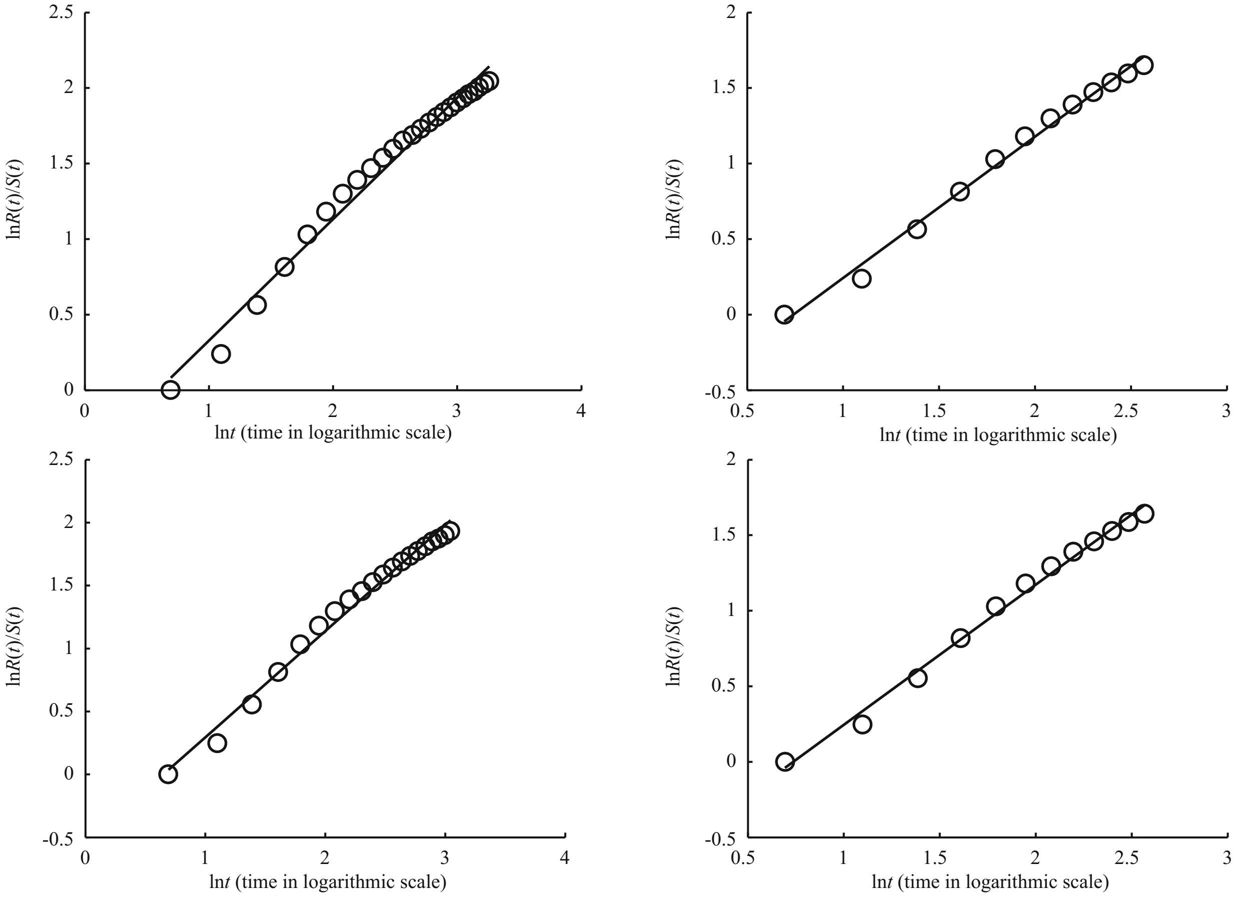

Fig.8 *R*/*S* analysis diagram of the different data

Table 5 Parameters for different distribution functions

Distribution function	Parameter	Interval
Gumbel	μ=41.237 2 σ=9.515 0	[36.887 5, 45.586 9] [7.165 8, 12.634 4]
Weibull	a=40.108 2 b=4.685 3	[36.445 7, 44.297 4] [3.522 1, 6.232 6]
Pearson-III	a=35.456 3 b=1.047	[19.419 3, 64.737 2] [0.571 2, 1.920 6]

From the data, we concluded that

(1) The parameters obtained from the long-term data and the short-term data were slightly different when using the new model.

(2) The difference between the results based on data from July and August and those from the annual extreme values was less than 7%. This implies that our proposed model is sufficiently stable to predict annual extreme values by using wave height time series in July and August.

The annual extreme values for 26 years and the first 13 years, as well as the extreme values during the summer in July and August for 26 years and the first 13 years, are shown in logarithmic coordinates in Fig.8.

The parameters and the intervals of confidence level higher than 95% (Shi, 2006), by using Gumbel, Weibull and Pearson-III models, are shown in Table 5.

The distribution fitting functions for the Gumbel, Weibull, and Pearson type III models as well as the empirical distribution function for extreme wave heights are shown in Fig.9.

The figure shows that the fitting using models based on none of the Gumbel, Weibull, or Pearson type III distributions is satisfactory. Wave heights occurring once every 100, 200, 400, 500, 700, and 1 000 years, by using the Gumbel, Weibull, Pearson III, as well as our proposed self-affine fractal model were estimated and are summarized in Table 6.

Similarly, considering seasonal effects, the design wave heights obtained using extreme values in the summer are listed in Table 7:

The above table shows that there is a slight difference between the estimated design wave heights of specified return periods by using the self-affine fractal model and traditional models. Consider Pearson type III. The difference between the two models is less than 0.294, but the values generated by

Table 6 Design wave heights for different distributions

Return period	Wave height (m)			
	Gumbel	Weibull	Pearson-III	Self-affine fractal model
10	4.866	4.747	4.470	4.004
20	5.106	5.006	4.707	4.265
25	5.146	5.052	4.753	4.327
50	5.349	5.283	4.982	4.636
100	5.497	5.460	5.172	4.939
200	5.625	5.616	5.349	5.261
300	5.692	5.701	5.449	5.458
400	5.737	5.756	5.517	5.603
500	5.770	5.801	5.570	5.718
700	5.818	5.862	5.647	5.896
1000	5.867	5.925	5.728	6.091

Table 7 Design wave heights in summer for different distributions

Return period	Wave height (m)			
	Gumbel	Weibull	Pearson-III	Self-affine fractal model
10	4.917	4.800	4.531	4.209
20	5.167	5.078	4.795	4.587
25	5.295	5.232	4.960	4.723
30	5.421	5.376	5.103	4.894
50	5.466	5.430	5.164	4.931
100	5.577	5.566	5.316	5.023
200	5.710	5.734	5.516	5.222
300	5.781	5.827	5.628	5.424
400	5.827	5.888	5.705	5.572
500	5.862	5.935	5.764	5.690
700	5.912	6.002	5.852	5.872
1 000	5.963	6.070	5.943	6.072

our model are larger when the return period increases, which means that the design wave heights obtained using the self-affine fractal model are more rigorous. Moreover, the difference between the design wave heights during summer between the Pearson type III distribution model and our novel model is less than 5.5%, as shown in Table 5. Furthermore, the fitting results of extreme wave heights by using traditional models are not satisfactory according to Fig.8. Therefore, our self-affine fractal model provides a new and effective approach to estimate design wave heights of specified return periods that is more accurate.

So far, the collected data was largely wasted, because the calculations of appropriate design parameters in practices such as ocean engineering, hydraulic engineering, nuclear power plant, etc., only use annual extremes. However, we have found that when the random variable is greater than a certain value, the distribution a similar with the power function. So we introduced the self-similarity and scale-invariance, and proposed a new model based on fractal theory. The new model considers the information of both extreme values and values over threshold, meanwhile it can also reflect their common characteristics. Our method can be used for calculating wave heights with a return period of 1 000 years for coastal nuclear power plants.

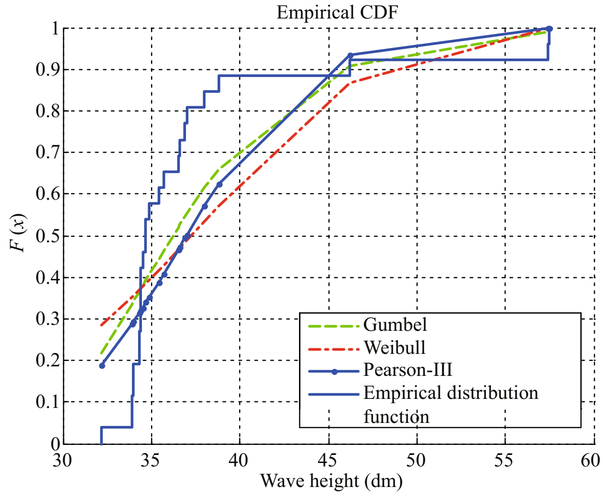

Fig.9 Fitting functions of Gumbel, Weibull, Pearson III models, empirical for extreme wave heights

4 CONCLUDING REMARKS

(1) After analyzing the in-situ wave height records along the coast of Chaolian Island from 1963 to 1989, we obtained the design wave heights of the specified return period by using *R*/*S* analysis and a self-affine fractal model. Comparing these values with those obtained using traditional models, we found that the design wave heights occurring once every 100 and 200 years by using the new model were smaller. The design wave heights occurring once every 300, 400, and 500 years were comparable, i.e., the design wave heights were higher than the Gumbel and Weibull distributions, but lower than the Pearson type III distribution model. The design wave heights occurring once every 700 and 1 000 years were larger. This indicated that our self-affine fractal model is more rigorous than traditional models for estimating design wave heights with large return periods. On the contrary, traditional single-factor models tend to underestimate design wave heights, which is risky for

coastal structures. This is because traditional methods prescribe distributions that are transcendent and subjective. On the other hand, *R*/*S* analysis assesses wave heights data and assures self-affinity in advance, following which design wave heights are estimated using a self-affine fractal model. This provides a new approach to predict design wave heights of specified return periods.

(2) Considering the geographical and seasonal effects on wave heights, such as typhoons, the extreme values recorded in July and August were analyzed and compared with annual extreme values. We found that the design wave heights during the summer were smaller when considering a return period of 100 years, but the difference was less than 1.7%. Thus, attention should be paid to long-term observations and measurements in July and August in the future, as this would be more accurate.

(3) When applying the self-affine fractal model to estimate wave heights of specified return periods, there was only a slight difference between results based on the short-term data and those based on long-term data. This further verifies the stabilities of our proposed model.

(4) Although fractal theories and *R*/*S* analysis were used, there remain unanswered issue, such as a method to minimize unpredictable parts occurring because of randomness and errors in observation, and whether the mean water level can be taken into account when applying *R*/*S* analysis, in order to eliminate uncertainties and subjectivity when using traditional methods and prescribing distribution by experience.

(5) Although the new fractal model is proposed by analyzing the oceanic topography, the self-similarity and scale-invariance of the measured tide data, it can be used to calculate the design water levels with a return period of 50 years, 100 years or higher, which include design highest water level, design lowest water level, check highest water level and check lowest water level.

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